

Package ‘afpt’

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Title Tools for Modelling of Animal Flight Performance

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Description Allows estimation and modelling of flight costs in animal (vertebrate) flight, implementing the aerodynamic power model described in Klein Heerenbrink et al. (2015) <[doi:10.1098/rspa.2014.0952](https://doi.org/10.1098/rspa.2014.0952)>. Taking inspiration from the program 'Flight', developed by Colin Pennycuick (Pennycuick (2008) "Modelling the flying bird". Amsterdam: Elsevier. ISBN 0-19-857721-4), flight performance is estimated based on basic morphological measurements such as body mass, wingspan and wing area. 'afpt' can be used to make predictions on how animals should adjust their flight behaviour and wingbeat kinematics to varying flight conditions.

URL <https://github.com/MarcoKlH/afpt-r/>

BugReports <https://github.com/MarcoKlH/afpt-r/issues>

Depends R(>= 4.0.0)

Imports graphics, stats

Suggests knitr, rmarkdown, testthat

VignetteBuilder knitr

License GPL (>= 3)

NeedsCompilation no

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R topics documented:

air2ground	2
altitude2density	3
amplitude	4
Bird	5
climbing_birds	7
computeAvailablePower	8
computeBodyFrontalArea	10
computeChemicalPower	11
computeFlappingPower	12
computeFlightPerformance	16
fDfPfunctions	19
findMaximumClimbRate	21
findMaximumPowerSpeed	22
findMaximumRangeSpeed	24
findMinimumPowerSpeed	25
findMinimumTimeSpeed	26
PowerToFroMechChem	28
reducedFrequency	29
Index	31

air2ground	<i>Compute groundspeed</i>
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Description

Computes groundspeed from airspeed and wind.

Usage

```
air2ground(airSpeed, windSpeed = 0, windDir = 0, climbAngle = 0)
```

Arguments

airSpeed	airspeed
windSpeed	windspeed
windDir	wind direction relative to (intended) track direction in degrees
climbAngle	climb angle in degrees

Value

driftAngle	Angle between airspeed and groundspeed
groundSpeed	Speed over ground

Author(s)

Marco Klein Heerenbrink

altitude2density *Compute density in International Standard Atmosphere*

Description

This function computes the air density at a specified altitude in the Troposphere of the International Standard Atmosphere.

Usage

```
altitude2density(altitude = 0)
```

Arguments

altitude (geopotential) altitude in meters above sealevel.

Details

$\rho = \rho_0 \left(1 + a \frac{h}{T_0}\right)^{-\frac{g_0}{R a} + 1}$ with $\rho_0 = 1.225$ kg/m³, $a = -0.0065$ K/m, h geopotential altitude in meters, $g_0 = 9.80665$ m/s², and $R = 287.1$ J/Kg/K.

Value

Numerical value or array for the density in kg/m³

Author(s)

M. Klein Heerenbrink

References

U.S. Standard Atmosphere, 1976, U.S. Government Printing Office, Washington, D.C.

Examples

```
altitude <- seq(0,3000,100) # meters above sealevel
density <- altitude2density(altitude)
```

amplitude

Flapping flight optimal amplitude

Description

This function returns the angular peak amplitude of the flapping motion, optimized for minimum induced power for prescribed reduced frequency (k_f), strokeplane angle (ϕ), and thrust-to-lift ratio (TL).

Usage

```
amplitude(kf, phi, TL)
```

Arguments

Using f for wingbeat frequency, b for wingspan, and U for air speed:

k_f	reduced frequency ($k_f = \frac{2\pi f b}{U}$); valid range between 1 and 6
ϕ	strokeplane angle in radians; valid range between 0 and 0.87 rad (50 deg)
TL	thrust requirement or the trust-to-lift ratio; valid range between 0 and 0.3

Value

Angular peak amplitude of the flapping motion in degrees.

Author(s)

Marco Klein Heerenbrink

References

Klein Heerenbrink, M., Johansson, L. C. and Hedenström, A. 2015 Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**, 2177 doi:10.1098/rspa.2014.0952

See Also

[computeFlappingPower](#)

Examples

```
## reduced frequency
kf <- 2*pi*4/10 # 4 Hz at 10 m/s for 1m wing span
## strokeplane angle
phi <- 20*pi/180 # 20 degrees
## thrust ratio
TL <- 0.2
## wingbeat amplitude
```

```
theta <- amplitude(kf,phi,TL)
print(theta)
# [1] 49.17679
```

 Bird

Bird description

Description

This function creates a bird description object, which is basically just a list with predefined variable names. It is named a bird object, but could also contain a description of a bat or insect. Minimal input required to construct a bird are body mass, wing span and wing area (or wing aspect ratio). Other required variables will then be given default values, or they will be estimated from allometric relations from literature.

Usage

```
Bird(massTotal, wingSpan, wingArea, ...)
```

Arguments

massTotal	Total mass that needs to be lifted in flight in kg
wingSpan	The maximum distance between the wingtips in meters
wingArea	The area of the fully stretched wings including the root area (left wing, right wing and area in between the wing roots)
...	Any other properties of a valid bird object (see details)

Details

This function sets up a list of properties of a bird. This definition of the bird is then used by the other functions in the package to estimate flight performance. At least three properties need to be specified: `massTotal`, `wingSpan` and `wingArea`. Either `wingSpan` or `wingArea` could be replaced by `aspectRatio`; the missing variable will then be computed. If no other properties are specified, default values will be used. Wingspan and wingarea should be measured from the maximally stretched out wing as described in *Pennycook (2008)*: wingspan as the maximum distance between the wingtips and wingarea as the area from a trace including the root area (where the body is).

To specify custom properties, these can simply be added as additional arguments to the function. Note that `massTotal` needs to be the sum of `massLoad`, `massFat` and `massEmpty`. The function will recompute the total mass if the specified masses are inconsistent. Allometric relations use the empty weight. Muscle mass is part of the empty mass, and as such it is represented by `muscleMass` as a fraction. It is used in the estimation of the mechanical power available for flight (together with the muscle properties `coef.activeStrain` and `coef.isometricStress`). The variable `type` is used for selected allometric relationships that are specific to that particular group. Currently, `bodyFrontalArea` distinguishes between 'passerine' and anything else and `basalMetabolicRate` distinguishes between 'passerine', 'seabird', 'bat' and anything else.

name	String	Common name
------	--------	-------------

name.scientific	String	Scientific name
source	String	Source for information
massLoad	Numeric	Additional mass the bird is carrying (kg); 0
massFat	Numeric	Fat mass, i.e. fuel (kg); 0
massEmpty	Numeric	Empty mass, i.e. total mass - fat mass - load mass (kg)
muscleFraction	Numeric	Fraction [0,1] of empty mass that makes up flight muscle; 0.17*
type	String	Type of bird 'other'*, 'passerine'*, 'seabird', 'bat'
bodyFrontalArea	Numeric	Reference body frontal area used for body drag (m ²)
wingbeatFrequency	Numeric	Typical wingbeat frequency (Hz)
coef.profileDragLiftFactor	Numeric	Coefficient for lift dependent profile drag; 0.03 (Klein Heerenbrink et al. 2015)
coef.bodyDragCoefficient	Numeric	Drag coefficient related to body frontal area; 0.2**
coef.conversionEfficiency	Numeric	Efficiency Chemical to Mechanical energy; 0.23*
coef.respirationFactor	Numeric	Multiplier for metabolic overhead respiration; 1.1*
coef.activeStrain	Numeric	Muscle duty cycle factor; 0.26*
coef.isometricStress	Numeric	Maximum force produced per cross section muscle (Pa); 400000 (upper limit for birds)
basalMetabolicRate	Numeric	Minimum energy consumption required for sustain life functions (W) *.

* as in Flight 1.25 (Pennycuick 2008)

** Large body of data supporting higher body drag coefficients (>0.2) than in Flight 1.25 (0.1), e.g. Pennycuick et al. (1988), Hedenström & Liechti (2001), Henningson & Hedenström (2011) and KleinHeerenbrink et al. (2016)

Value

bird object with variables required by the various power estimating functions (e.g. `computeFlappingPower`).

Author(s)

Marco Klein Heerenbrink

References

- Hedenström, A. & Liechti, F. (2001) Field estimates of body drag coefficient on the basis of dives in passerine birds. *J. Exp. Biol.* **204**, 1167–75.
- Henningson, P. & Hedenström, A. (2011) Aerodynamics of gliding flight in common swifts. *J. Exp. Biol.* **214**, 382–93. doi:10.1242/jeb.050609
- Klein Heerenbrink, M., Johansson, L. C. & Hedenström, A. (2015) Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**. doi:10.1098/rspa.2014.0952
- KleinHeerenbrink, M., Warfvinge, K. & Hedenström, A. (2016) Wake analysis of aerodynamic components for the glide envelope of a jackdaw (*Corvus monedula*). *J. Exp. Biol.* **219**, 1572–1581. doi:10.1242/jeb.132480
- Pennycuick, C. J. & Rezende, M. A. (1984) The specific power output of aerobic muscle, related to the power density of mitochondria. *J. Exp. Biol.*, **108**, 377–392.
- Pennycuick, C. J., Obrecht III, H. H. & Fuller, M. R. (1988) Empirical estimates of body drag of large waterfowl and raptors. *J. Exp. Biol.* **135**, 253–264.
- Pennycuick, C. J. (2008). *Modelling the flying bird*. Amsterdam, The Netherlands: Elsevier.

See Also

[computeAvailablePower](#), [computeChemicalPower](#), [computeFlappingPower](#), [computeBodyFrontalArea](#), etc.

Examples

```
myBird = Bird(  
  massTotal = 0.215,  
  wingSpan = 0.67,  
  wingArea = 0.0652,  
  name = 'jackdaw',  
  type = 'passerine'  
)  
print(myBird)
```

climbing_birds

Climbing birds

Description

Data extracted from *Hedenström & Alerstam 1992*.

Usage

```
data("climbing_birds")
```

Format

A data frame with 15 observations on the following 11 variables.

number a numeric vector

name a character vector

name.scientific a character vector

massEmpty a numeric vector

massFat a numeric vector

wingSpan a numeric vector

wingAspect a numeric vector

wingbeatFrequency a numeric vector

climbRate a numeric vector

climbSpeed a numeric vector

climbAltitude a numeric vector

Source

Hedenström A., Ålerstam, T. (1992) Climbing performance of migrating birds as a basis for estimating limits for fuel-carrying capacity and muscle work. *J. Exp. Biol* **164** 19-38 doi:10.1242/jeb.164.1.19

Examples

```
data(climbing_birds)
climbingBirds <- Bird(climbing_birds)
```

computeAvailablePower *Compute available power*

Description

Estimation of maximum available power available from the muscles.

Usage

```
computeAvailablePower(bird, maxPowerAero, ...)
```

Arguments

bird	bird description object (see Bird)
maxPowerAero	maximum continuous power
...	optional arguments (none yet)

Details

Available power is determined as a muscle property. It is assumed that part of the muscles tissue is chemically active (mitochondria), providing the required ATP energy to the mechanically active tissue (myofibrils). The fraction of mitochondria determines the maximum sustainable power output from the muscles. With a higher fraction of myofibrils, the muscles can produce more power, but only in a short burst, until all ATP runs out.

If only a [Bird](#) object is provided, the function will assume that maximum power equals maximum continuous power (maxPowerAero). Otherwise, it will compute the burst maximum power.

Value

numeric value of mechanical power

Note

Available power is determined as a constant for the muscles. In reality the muscle power output depends on strainrate and stress, which in vertebrates are directly linked to wingbeat kinematics and aerodynamic loads.

Flight 1.25, the model of *Pennycuick (2008)* uses an isometric stress of 560 kN/m². This is much higher than any measured value (*Pennycuick & Rezende 1984*). A more reasonable yet still very optimistic value would be 400 kN/m², which is the default value assigned by the [Bird](#) constructor.

Author(s)

Marco Klein Heerenbrink

References

Pennycuick, C. J. & Rezende, M. A. (1984) The specific power output of aerobic muscle, related to the power density of mitochondria. *J. Exp. Biol.*, **108**, 377–392.

Pennycuick, C. J. (2008). *Modelling the flying bird*. Amsterdam, The Netherlands: Elsevier.

See Also

[Bird](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

## for maximum continuous power
power.max <- computeAvailablePower(myBird)
print(power.max)
# [1] 5.233528

## for specified maximum continuous power:
power.max.continuous <- 0.8*power.max
power.max.burst <- computeAvailablePower(myBird,power.max.continuous)
print(power.max.burst)
# [1] 5.466625
```

`computeBodyFrontalArea`*Body frontal area from scaling relation*

Description

Body frontal area is a parameter that relates to body drag. This function estimates body frontal area based on empirical scaling relations with mass.

Usage

```
computeBodyFrontalArea(massEmpty, type = "other")
```

Arguments

<code>massEmpty</code>	empty body mass (in kg)
<code>type</code>	type of bird; available options are: "passerine" and "other")

Details

Passerine (*Hedenström and Rosén 2003*): $S_b = 0.0129m^{0.614}$

Other (*Pennycuick et al. 1988*): $S_b = 0.00813m^{0.666}$

Value

Numeric value for the body frontal area.

Note

Body frontal area is used for the computation of body drag. Only use this value if it matches the used definition of the body drag coefficient.

Author(s)

Marco Klein Heerenbrink

References

Pennycuick, C. J., Obrecht III, H. H. and Fuller, M. R. (1988) Empirical estimates of body drag of large waterfowl and raptors. *J. Exp. Biol.* **135**, 253–264.

Hedenström, A. and Rosén, M. (2003) Body frontal area in passerine birds. *J. Avian Biol.* **34**, 159–162.

See Also

[Bird](#)

Examples

```

massEmpty <- 0.215 # kg
Sb <- computeBodyFrontalArea(massEmpty)
print(Sb)
# [1] 0.002920751 # m2

massEmpty <- 0.215 # kg
birdType <- "passerine" #
Sb <- computeBodyFrontalArea(massEmpty,birdType)
print(Sb)
# [1] 0.005020037 # m2

```

```
computeChemicalPower Convert mechanical power to chemical power
```

Description

Redundant after chemical power is now computed in all functions by default.

Computes the chemical power, i.e. the rate at which chemical energy is consumed, during flight. It takes into account the basal metabolic rate, and the energy needed by the flight muscles to provide the mechanical power required for flight.

Usage

```

## S3 method for class 'power.mechanical'
computeChemicalPower(power.mech, bird, ...)
## S3 method for class 'numeric'
computeChemicalPower(power.mech, bird, ...)

```

Arguments

power.mech	mechanical power (either numeric (W) or as an mechanical power object (class power.mechanical))
bird	object describing the relevant morphological parameters of the bird (or bat); this object should be created using the Bird constructor.
...	optional arguments (none yet)

Details

Chemical power is computed as

$$P_{\text{chem}} = R \left(\frac{P_{\text{mech}}}{\eta} + \text{BMR} \right)$$

as described by *Pennyquick (2008)*. Here R is the respiration factor, η is the muscle conversion efficiency and BMR the basal metabolic rate, see [Bird](#).

Value

Chemical power of same type as inpute power . chem.

Author(s)

Marco Klein Heerenbrink

References

Pennyquick, C. J. (2008). *Modelling the flying bird*. Amsterdam, The Netherlands: Elsevier.

See Also

[Bird](#), [computeFlappingPower](#), [mech2chem](#), [chem2mech](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

## for maximum continuous power
power.max <- computeAvailablePower(myBird)
print(power.max)
# [1] 5.233528

## convert to chemical power
power.max.chem <- computeChemicalPower(power.max,myBird)
print(power.max.chem)
# [1] 27.28913
```

computeFlappingPower *Calculate aerodynamic power flapping flight*

Description

The function calculates the aerodynamic power required for the specified bird (or bat) at the specified flight speed.

Usage

```
computeFlappingPower(bird,speed,...,frequency,strokeplane)
```

Arguments

bird	object describing the relevant morphological parameters of the bird (or bat); this object should be created using the <code>Bird</code> constructor.
speed	a numeric vector of the airspeed.
...	optional arguments (see details)
frequency	wingbeat frequency as single numeric value, a numeric vector matching the speed vector, a closure object returning a numeric value as a function of speed, or the character string 'recompute'. The latter will recompute the default frequency for the current flight condition (density) and the current total mass of the bird (assuming the frequency in bird is the default wingbeat frequency). If not provided, the function will look for a default wingbeat frequency in the bird object.
strokeplane	angle of the strokeplane in degrees, as a single numeric value, a numeric vector matching the speed vector, a closure object describing the strokeplane angle as a function of speed. Alternatively providing character string "opt" will tell the function to optimize the strokeplane angle for minimum aerodynamic power.

Details

This function estimates aerodynamic power for an animal in forward flight based on morphology and wingbeat kinematics (*Klein Heerenbrink, 2015*). The model takes into account span reduction during the upstroke, which is typical for vertebrate forward flight. . . . The minimal input required for the function is a description of the animal (as provided by the `Bird` constructor) and the speed(range) for which to compute the aerodynamic power. Distinct from other models, this model also requires wingbeat frequency and strokeplane angle. Higher wingbeat frequency tends to lower the induced power, but it may increase profile power. If no wingbeat frequency is provided, the function will use the reference wingbeat frequency from the bird object. Otherwise the user can specify values (either as vectors or as closure object). The user can provide additional optional arguments:

`bodyDragCoefficient` single numeric value, a numeric vector matching the speed vector, or a closure object as a function of speed. If not provided, the function will look for a default value in the bird object.

`addedDrag` single numeric value or a numeric vector matching the speed vector. This represents additional "drag" (in Newtons) that must be overcome (e.g. during climb).

`flightcondition` object describing the atmospheric conditions (density, viscosity, gravity).

Aerodynamic model: `computeFlappingPower` first computes the drag components for non-flapping flight:

$$D_{\text{ind}} = \frac{L^2}{q\pi b^2}$$

$$D_{\text{pro},0} = C_{D_{\text{pro},0}} qS$$

$$D_{\text{pro},2} = k_p \frac{L^2}{qS}$$

$$D_{\text{par}} = C_{D_b} qS_b + D_{\text{added}}$$

which combine to the non-flapping thrust requirement $T_0 = \sum D_{\langle \rangle}$. Here $q = \frac{1}{2}\rho U^2$ is the dynamic pressure depending on density (ρ) and speed (U). To account for how flapping the

wings affects the drag on the wings, computeFlappingPower computes factors $f_{D_{\text{ind}}}$, $f_{D_{\text{pro},0}}$ and $f_{D_{\text{pro},2}}$, which are functions of the strokeplane angle and the (reduced) wingbeat frequency. These factors relate to the returned drag factors `kD.ind`, `kD.pro0` and `kD.pro2` through

$$k_{D,<>} = 1 + f_{D,<>} \frac{T}{L}$$

The actual drag in flapping flight is found by multiplying each non-flapping drag component with its respective drag factor. This means that the actual thrust requirement (thrust ratio T/L) can be computed as

$$\frac{T}{L} = \frac{T_0}{L - f_{D_{\text{ind}}} D_{\text{ind}} - f_{D_{\text{pro},0}} D_{\text{pro},0} - f_{D_{\text{pro},2}} D_{\text{pro},2}}$$

Finally, computeFlappingPower computes the power factors in a similar way to the drag factors (i.e. $k_{P,i} = 1 + f_{P,i} \frac{T}{L}$, with $f_{P,i}$ functions of strokeplane angle and wingbeat frequency). The total aerodynamic power is then computed as

$$P = k_{P_{\text{ind}}} D_{\text{ind}} U + k_{P_{\text{pro},0}} D_{\text{pro},0} U + k_{P_{\text{pro},2}} D_{\text{pro},2} U + D_{\text{par}} U$$

Wingbeat optimization: The underlying numerical model that is represented by functions $f_{D,i}$ and $f_{P,i}$, has optimised the flapping amplitude for minimum induced power. This means computeFlappingPower implicitly optimizes flapping amplitude, which is the value `amplitude` returned in the output.

computeFlappingPower takes strokeplane angle as input. The underlying numerical model has only explored strokeplane angles over a range of 0 (vertical) to 50 degrees, the latter being defined as having the down-stroke moving forward. In many cases it will be possible to find a strokeplane angle for which the total aerodynamic power is minimal. At high speeds this optimum will be for a vertical strokeplane while at lower speeds it will be more horizontal. By passing `strokeplane="opt"` as an argument to computeFlappingPower, it will try to numerically find the optimal strokeplane angle, using the function `optimize`.

Value

A data.frame including elements

<code>speed</code>	specified speed for which power is computed.
<code>power</code>	total aerodynamic power.
<code>power.chem</code>	total chemical power.
<code>strokeplane</code>	used strokeplane angle (either specified or optimized).
<code>amplitude</code>	wingbeat amplitude (implicitly optimized for minimum induced power).
<code>frequency</code>	wingbeat frequency (specified).
<code>flags.redFreqLo</code>	TRUE if reduced frequency too low (<1; outside model range).
<code>flags.redFreqHi</code>	TRUE if reduced frequency too high (>6; outside model range).
<code>flags.thrustHi</code>	TRUE if thrust requirement too high (>0.3; outside model range).
<code>flags.speedLo</code>	TRUE if speed is too low (invalidating the forward flight assumption).
<code>kD.ind</code>	induced drag factor

kD.pro0	zero lift profile drag factor
kD.pro2	lift dependent profile drag factor
kP.ind	induced power factor
kP.pro0	zero lift profile power factor
kP.pro2	lift dependent profile power factor
CDpro0	used zero lift profile drag coefficient (laminar boundary layer friction)
ReynoldsNumber	mean chord Reynolds number
Dnf.ind	non-flapping induced drag (N)
Dnf.pro0	non-flapping zero lift profile drag (N)
Dnf.pro2	non-flapping lift dependent profile drag (N)
Dnf.par	non-flapping parasitic drag (including body drag and apparent drag due to climbing)
L	lift (N)

Note

This model aims to predict the optimal flight performance for a bird. Particularly, the induced drag and induced power assume an ideal load distribution over the wing equivalent to the elliptical lift distribution for non-flapping wings. This means that induced power will typically be underestimated.

Author(s)

Marco Klein Heerenbrink

References

Klein Heerenbrink, M., Johansson, L. C. and Hedenström, A. (2015) Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**, 2177 doi:[10.1098/rspa.2014.0952](https://doi.org/10.1098/rspa.2014.0952)

See Also

[Bird](#), [amplitude](#), [fD.ind](#), [fD.pro0](#), [fD.pro2](#), [fP.ind](#), [fP.pro0](#), [fP.pro2](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

## define a speed range
speedrange <- seq(5,14,length.out=5)
```

```

## compute aerodynamic power for that speed range:
Paero <- computeFlappingPower(myBird,speedrange)
print(Paero[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 5.00 2.789751 5.948083 46.56887
# 2 7.25 2.129466 5.948083 31.89129
# 3 9.50 2.203773 5.948083 22.51896
# 4 11.75 2.740763 5.948083 16.49120
# 5 14.00 3.673714 5.948083 12.09174

## prescribe strokeplane angle:
Paero <- computeFlappingPower(myBird,speedrange,strokeplane=20)
print(Paero[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 5.00 2.950259 5.948083 20
# 2 7.25 2.141581 5.948083 20
# 3 9.50 2.204132 5.948083 20
# 4 11.75 2.741335 5.948083 20
# 5 14.00 3.676224 5.948083 20

## prescribe frequency as a function of speed:
funFrequency = function(U){19.8 - 4.7*U + 0.45*U^2 - 0.0138*U^3}
Paero <- computeFlappingPower(myBird,speedrange,frequency=funFrequency,strokeplane='opt')
print(Paero[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 5.00 2.810431 5.825000 46.16223
# 2 7.25 2.356278 4.119247 25.99702
# 3 9.50 2.390251 3.930725 17.94304
# 4 11.75 2.860463 4.316291 14.52910
# 5 14.00 3.794431 4.332800 11.70058

## examine effect of frequency for a single airspeed:
speedrange <- rep(10,5) # repeated speed
freqrange <- seq(3,10,length.out=5) # frequency range
Paero <- computeFlappingPower(myBird,speedrange,frequency=freqrange,strokeplane='opt')
print(Paero[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 10 2.681028 3.00 13.87797
# 2 10 2.367982 4.75 18.90949
# 3 10 2.263765 6.50 21.52433
# 4 10 2.219739 8.25 21.71519
# 5 10 2.200852 10.00 20.18503

```

computeFlightPerformance

Compute characteristics of a power curve

Description

This function calculates the basic characteristic flight speeds for bird.

Usage

```
computeFlightPerformance(bird, ..., length.out=10)
```

Arguments

bird	description of the bird or bat, constructed using the Bird function
...	various optional arguments that are passed on to other functions; see details
length.out	length of calculated power curve; set length.out=0 to not compute a power curve

Details

Optional arguments can be provided through ... These can be arguments of computeFlappingPower, e.g. strokeplane, frequency, etc., or arguments for findMaximumRangeSpeed, e.g. windSpeed and windDir. The latter will only affect the outcome of the maximum range speed, and should perhaps not be analysed through the current function...

Value

birdWSName	variable name in work-space of the bird object
bird	bird object
table	table with characteristic speeds
maxClimb	table with climb performance
powercurve	power curve from minimum to maximum speed of length length.out

Author(s)

Marco Klein Heerenbrink

References

Klein Heerenbrink, M., Johansson, L. C. and Hedenström, A. (2015) Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**, 2177 doi:[10.1098/rspa.2014.0952](https://doi.org/10.1098/rspa.2014.0952)

See Also

[Bird](#), [computeFlappingPower](#)

Examples

```
## Not run: # computationally intensive

## Define a bird:
myBird = Bird(
  name = "Jackdaw",
  name.scientific = "Corvus monedula",
  massTotal = 0.215, # (kg) total body mass
```

```

    wingSpan = 0.67, # (m) maximum wing span
    wingArea = 0.0652, # (m2) maximum wing area
    type = "passerine"
)

## simplest performance calculation
performance.myBird <- computeFlightPerformance(myBird)
performance.myBird
# Name: Jackdaw
# Sc. name: Corvus monedula
# Bird definitions: NA
#           speed power.aero power.chem strokeplane amplitude
# minimumSpeed 2.706      5.234      27.29      49.9      51.3
# minimumPower 8.031      2.093      12.27      28.1      34.5
# maximumRange 11.025     2.523      14.33      18.2      36.7
# maximumSpeed 16.590     5.235      27.29      6.8       50.2
# Maximum climb performance:
#           speed power.aero power.chem strokeplane amplitude climbRate
# maximumClimbRate 8.89      5.234      27.29      24.5      53.9      1.18
# Minimized migration time:
#           speed speed.migration power.aero power.chem power.dep strokeplane amplitude
# minimumTimeSpeed 11.75      1.962      2.741      15.37     3.081      16.49     38.04

## optimize strokeplane angle and use speed dependent frequency
funFrequency = function(U){19.8 - 4.7*U + 0.45*U^2 - 0.0138*U^3}
performance.myBird <- computeFlightPerformance(myBird,strokeplane='opt',frequency=funFrequency)
performance.myBird
# Name: Jackdaw
# Sc. name: Corvus monedula
# Bird definitions: NA
#           speed power.aero power.chem strokeplane amplitude
# minimumSpeed 2.293      5.229      27.27      49.9      43.8
# minimumPower 8.192      2.319      13.35      21.6      42.8
# maximumRange 11.463     2.775      15.53      14.9      44.3
# maximumSpeed 16.088     5.233      27.29      8.3       64.5
# Maximum climb performance:
#           speed power.aero power.chem strokeplane amplitude climbRate
# maximumClimbRate 8.89      5.234      27.29      24.5      53.9      1.18
# Minimized migration time:
#           speed speed.migration power.aero power.chem power.dep strokeplane amplitude
# minimumTimeSpeed 12.07      1.905      2.964      16.43     3.081      14.13     45.13

## plot variation of speed, power and flapping kinematics
plot(performance.myBird$powercurve[c('speed','power.aero','strokeplane','frequency','amplitude')])

## plot power factors
plot(performance.myBird$powercurve[c('speed','power.aero')])
plot(performance.myBird$powercurve[c('speed','kP.ind')])
plot(performance.myBird$powercurve[c('speed','kP.pro0')])
plot(performance.myBird$powercurve[c('speed','kP.pro2')])

## End(Not run) # end dontrun

```

fDfPfunctions

*Coefficient for thrust dependency of drag and power factors***Description**

Computes the thrust requirement dependency factor for drag and power factors in flapping flight based on reduced frequency (k_f) and strokeplane angle (ϕ).

Usage

```
fD.ind(kf, phi)
fD.pro0(kf, phi)
fD.pro2(kf, phi)
fP.ind(kf, phi)
fP.pro0(kf, phi)
fP.pro2(kf, phi)
```

Arguments

Using f for wingbeat frequency, b for wingspan, and U for air speed:

k_f reduced frequency ($k_f = \frac{2\pi fb}{U}$); valid range between 1 and 6
 ϕ strokeplane angle in radians; valid range between 0 and 0.87 rad (50 deg)

Details

Flapping of the wings alters the drag components on the wing. A drag component in flapping flight can be related to the drag component in non-flapping flight as $D = k_D D'$. The factor k_D depends on reduced frequency k_f , strokeplane angle ϕ and the thrust-to-lift ratio T/L : $k_D = 1 + f_D(k_f, \phi) \frac{T}{L}$. Functions `fD.ind`, `fD.pro0` and `fD.pro2` compute $f_D(k_f, \phi)$ for induced drag, zero lift profile drag and lift dependent profile drag, respectively.

Similarly, the flapping power components can be computed as: $P = k_P D' U$, again with $k_P = 1 + f_P(k_f, \phi) \frac{T}{L}$. Functions `fP.ind`, `fP.pro0` and `fP.pro2` compute $f_P(k_f, \phi)$ for induced power, zero lift profile power and lift dependent profile power, respectively.

Value

Numeric value

Note

Thrust requirement is the sum of all drag components in flapping flight divided by the lift. This means the thrust requirement itself is a function of the values of f_D .

Author(s)

Marco Klein Heerenbrink

References

Klein Heerenbrink, M., Johansson, L. C. and Hedenström, A. 2015 Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**, 2177 doi:10.1098/rspa.2014.0952

See Also

[computeFlappingPower](#)

Examples

```
## reduced frequency
kf <- 2*pi*4/10 # 4 Hz at 10 m/s

## strokeplane angle
phi <- 20*pi/180 # 20 degrees

## thrust ratio
TL <- 0.2

## induced drag factor:
fDind <- fD.ind(kf,phi)
kDind <- 1 + fDind*TL
print(kDind)
# [1] 1.623659

## zero lift drag factor:
fDpro0 <- fD.pro0(kf,phi)
kDpro0 <- 1 + fDpro0*TL
print(kDpro0)
# [1] 1.014899

## lift dependent profile drag factor:
fDpro2 <- fD.pro2(kf,phi)
kDpro2 <- 1 + fDpro2*TL
print(kDpro2)
# [1] 1.511107

## induced power factor:
fPind <- fP.ind(kf,phi)
kPind <- 1 + fPind*TL
print(kPind)
# [1] 1.996891

## zero lift power factor:
fPpro0 <- fP.pro0(kf,phi)
kPpro0 <- 1 + fPpro0*TL
print(kPpro0)
# [1] 1.076046

## lift dependent profile power factor:
fPpro2 <- fP.pro2(kf,phi)
```

```
kPpro2 <- 1 + fPpro2*TL
print(kPpro2)
# [1] 1.811983
```

findMaximumClimbRate *Find maximum climb rate*

Description

Numerically find the maximum attainable climb rate.

Usage

```
findMaximumClimbRate(bird, maximumPower, speed, ...)
```

Arguments

bird	bird description object (see Bird)
maximumPower	numeric value for maximum available mechanical power
speed	airspeed for which to compute the maximum climb rate
...	optional arguments for computeFlappingPower

Details

The function searches for a climb angle between -90 and 90 degrees that matches the specified maximum power available. If no speed provided, the function will also find the optimal airspeed for maximum climb rate.

Value

Data frame of class `power.mechanical`

speed	airspeed either prescribed or optimized for maximum climb rate
power	aerodynamic (mechanical) power matching maximum power
...	see computeFlappingPower for other variables
climbAngle	angle between flightpath and horizontal plane in degrees
climbRate	rate of vertical climb

Note

The function uses climb angle, rather than climb rate, in the search algorithm, to ensure that climb rate is always less than the airspeed (i.e. in a vertical climb the climb rate will simply equal airspeed). The actual climb rate is maximized by maximizing the product of climb angle and airspeed. However, in practice, the airspeed for best climb rate will be close to the minimum power airspeed, where the power margin is largest.

Author(s)

Marco Klein Heerenbrink

See Also

[uniroot](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

## maximum power available:
Paero.available <- computeAvailablePower(myBird)

climbSpeed <- 8 # airspeed during climb

## find maximum climb rate:
Paero.climb <- findMaximumClimbRate(myBird, Paero.available, climbSpeed)
print(Paero.climb[c('speed', 'amplitude', 'frequency', 'climbRate')])
# speed amplitude frequency climbRate
# 1      8 54.84965 5.948083 1.162002
```

`findMaximumPowerSpeed` *Finds speed for which power required equals maximum available power*

Description

Numerically find the airspeed for which required power equals maximumPower.

Usage

```
findMaximumPowerSpeed(bird, maximumPower, lower, upper, ...)
```

Arguments

<code>bird</code>	bird description object (see Bird)
<code>maximumPower</code>	numeric value for maximum available mechanical power
<code>lower</code>	lower bound for search range airspeed (m/s)
<code>upper</code>	upper bound for search range airspeed (m/s)
<code>...</code>	optional arguments to computeFlappingPower

Details

Prepares arguments for a call to [uniroot](#). The function searches for an airspeed between lower and upper that matches the specified maximum power available.

Value

Data frame

speed	airspeed for which power matches maximum power
power	aerodynamic (mechanical) power matching maximum power
power.chem	aerodynamic (mechanical) power matching maximum power
strokeplane	optimized or prescribed strokeplane angle in degrees (from vertical)
amplitude	optimized peak amplitude in degrees (see amplitude)
...	see computeFlappingPower for other variables

Note

Typically this function would be used to find the maximum speed, but may in some cases also be used for the minimum flight speed. However, note that the low speed limit is likely limited by other constraints as well (e.g. stall speed).

Author(s)

Marco Klein Heerenbrink

See Also

[uniroot](#)

Examples

```
## Define a bird:
myBird <- Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

Paero.available <- computeAvailablePower(myBird)

## find maximum speed:
Vmin <- 5
Vmax <- 30
Paero.maxSpeed <- findMaximumPowerSpeed(myBird,Paero.available,Vmin,Vmax)
print(Paero.maxSpeed[c('speed','power','amplitude','strokeplane','frequency')])
#   speed   power amplitude strokeplane frequency
# 1 16.58797 5.233459 50.22762   6.812345 5.948083
```

findMaximumRangeSpeed *Find maximum range speed*

Description

This function performs a numerical optimization to find the airspeed for which $\frac{P}{U}$ is minimum. For this it uses the function `optimize`.

Usage

```
findMaximumRangeSpeed(bird, lower=NULL, upper=NULL, windSpeed=0, windDir=0, ...)
```

Arguments

<code>bird</code>	bird description object (see Bird)
<code>lower</code>	lower speed limit (optional)
<code>upper</code>	upper speed limit (optional)
<code>windSpeed</code>	wind magnitude (in m/s; optional)
<code>windDir</code>	wind direction (in degrees; optional)
<code>...</code>	optional arguments: <code>climbAngle</code> (in degrees), and optional arguments for computeFlappingPower .

Details

This function performs a numerical optimization to find the airspeed for which $\frac{P}{U}$ is minimum. For this it uses the function `optimize`. This airspeed is searched for between `lower` and `upper` (if not provided, it will make a guess based on `bird`). Flying in wind changes the ground speed, and therefore the optimum flight speed for maximum range. This can be taken into account through the optional arguments for wind magnitude (`windSpeed` in m/s) and wind direction relative to the track direction (`windDir` in degrees; `windDir = 0` tail wind); see e.g. *Liechti et al. 1994*.

Value

Returns `data.frame` (power.chemical) of flight performance at maximum range speed for `bird`.

Author(s)

Marco Klein Heerenbrink

References

Liechti, F., Hedenström, A. and Alerstam, T. (1994). Effects of Sidewinds on Optimal Flight Speed of Birds. *J. Theor. Biol.* **170**, 219–225.

See Also

[computeChemicalPower](#), [computeFlappingPower](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

maximumRangeSpeed.chem <- findMaximumRangeSpeed(myBird)
maximumRangeSpeed.chem[c('speed', 'power', 'strokeplane', 'amplitude', 'frequency')]
#   speed   power strokeplane amplitude frequency
# 1 11.02543 14.32754   18.17729  36.69311  5.948083

maximumRangeSpeed.chem.wind <- findMaximumRangeSpeed(
  myBird,
  windSpeed = 5,
  windDir = 90
)
maximumRangeSpeed.chem.wind[c('speed', 'power', 'strokeplane', 'amplitude', 'frequency')]
#   speed   power strokeplane amplitude frequency
# 1 11.81974 15.47758   16.33727  38.17508  5.948083
```

findMinimumPowerSpeed *Find speed for minimum power*

Description

.

Usage

```
findMinimumPowerSpeed(bird, lower, upper, ...)
```

Arguments

bird	bird description object (see Bird)
lower	lower speed limit (optional)
upper	upper speed limit (optional)
...	optional arguments for <code>computeFlappingPower()</code>

Details

This is pretty much just a call to `optimize`.

Value

powercurve object (funCalcPower evaluated for the minimum speed)

Author(s)

Marco Klein Heerenbink

See Also

[optimize](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

minimumPowerSpeed.aero <- findMinimumPowerSpeed(myBird)
minimumPowerSpeed.aero[c('speed', 'power', 'strokeplane', 'amplitude', 'frequency')]
#   speed   power strokeplane amplitude frequency
# 1 8.030022 2.092976   28.14514  34.52719  5.948083
```

findMinimumTimeSpeed *Find speed for migration time minimization*

Description

This function performs a numerical optimization to find the airspeed for which $\frac{P+P_{\text{dep}}}{U}$ is minimum..

Usage

```
findMinimumTimeSpeed(bird,
  EnergyDepositionRate=1.5*bird$basalMetabolicRate,
  lower=NULL, upper=NULL,
  windSpeed=0, windDir=0, ...)
```

Arguments

bird	bird description object (see Bird)
EnergyDepositionRate	The rate at which the bird accumulates energy at stopover sites
lower	lower speed limit (optional)
upper	upper speed limit (optional)
windSpeed	wind magnitude (in m/s; optional)
windDir	wind direction (in degrees; optional)
...	optional arguments: <code>climbAngle</code> (in degrees), and optional arguments for computeFlappingPower .

Details

This function performs a numerical optimization to find the airspeed that minimizes the combination of flight time and time required to (re)gain the energy reserves to cover the flight cost. If the bird would fly faster, it would need to spend more time refueling. If it flew slower, the reduced refueling time that comes with the lower cost of transport does not offset the longer flight time. Mathematically this problem works out as minimizing $\frac{P+P_{dep}}{U}$ Hedenström 1998, which is technically the same optimization as for the maximum range speed (see details [findMaximumRangeSpeed](#)). The default energy deposition rate, the rate at which a bird accumulates energy during a stopover, is set to 1.5 times the basal metabolic rate (Lindström 1991).

Value

Returns data.frame (power.chemical) of flight performance at maximum range speed for bird.

Author(s)

Marco Klein Heerenbrink

References

- Lindström, Å. (1991) Maximum fat deposition rates in migrating birds. *Ornis Scand.* **22**, 12-19 (doi:10.2307/3676616)
- Hedenström, A. & Ålerstam, T. (1997) Optimum fuel loads in migratory birds: distinguishing between time and energy minimization. *J. Theor. Biol.* **189**, 227–34. (doi:10.1006/jtbi.1997.0505)
- Hedenström, A. & Ålerstam, T. (1998) How fast can birds migrate? *J. Avian Biol.* **29**, 424-432. (doi:10.2307/3677161)

See Also

[computeChemicalPower](#), [computeFlappingPower](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

minimumTimeSpeed <- findMinimumTimeSpeed(myBird,1.5*myBird$basalMetabolicRate)
minimumTimeSpeed[c('speed','speed.migration',
  'power','power.chem','power.dep',
  'strokeplane','amplitude','frequency')]
# speed speed.migration power power.chem power.dep strokeplane amplitude frequency
# 11.74944 1.962213 2.74058 15.36634 3.080752 16.49244 38.03366 5.948083
```

PowerToFroMechChem *Convert between mechanical and chemical power*

Description

Functions convert between mechanical and chemical power

Usage

```
mech2chem(power.mech,bird,...)
chem2mech(power.chem,bird,...)
```

Arguments

power.mech	Numerical value for mechanical power
power.chem	Numerical value for chemical power
bird	object describing the relevant morphological parameters of the bird (or bat); this object should be created using the Bird constructor.
...	optional arguments (none yet)

Details

Chemical power is computed as

$$P_{\text{chem}} = R \left(\frac{P_{\text{mech}}}{\eta} + \text{BMR} \right)$$

as described in *Pennycuick 2008*. Here R is the respiration factor, η is the muscle conversion efficiency and BMR the basal metabolic rate, see [Bird](#).

Mechanical power is simply calculated inversely:

$$P_{\text{mech}} = \eta \left(\frac{P_{\text{chem}}}{R} - \text{BMR} \right)$$

Value

Numerical value of either chemical power (`mech2chem()`) or mechanical power (`chem2mech()`).

Author(s)

Marco Klein Heerenbrink

References

Pennycuick, C. J. (2008). *Modelling the flying bird*. Amsterdam, The Netherlands: Elsevier.

See Also

[computeChemicalPower](#)

Examples

```
## Define a bird:
myBird = Bird(
  massTotal = 0.215, # (kg) total body mass
  wingSpan = 0.67, # (m) maximum wing span
  wingArea = 0.0652, # (m2) maximum wing area
  type = "passerine"
)

## define a speed range
speedrange <- seq(5,14,length.out=5)

## compute aerodynamic power for that speed range:
Paero <- computeFlappingPower(myBird,speedrange)
Pchem <- Paero
Pchem$power <- mech2chem(Paero$power,myBird)
print(Pchem[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 5.00 15.60151 5.948083 46.56887
# 2 7.25 12.44362 5.948083 31.89129
# 3 9.50 12.79900 5.948083 22.51896
# 4 11.75 15.36721 5.948083 16.49120
# 5 14.00 19.82915 5.948083 12.09174

Pmech <- Pchem
Pmech$power <- chem2mech(Pchem$power,myBird)
print(Pmech[c("speed","power","frequency","strokeplane")])
# speed power frequency strokeplane
# 1 5.00 2.789751 5.948083 46.56887
# 2 7.25 2.129466 5.948083 31.89129
# 3 9.50 2.203773 5.948083 22.51896
# 4 11.75 2.740763 5.948083 16.49120
# 5 14.00 3.673714 5.948083 12.09174
```

reducedFrequency *Function to compute reduced frequency*

Description

This function computes the reduced frequency based on wingSpan (b), wingbeat frequency (f) and speed (U): $k_f = \frac{2\pi b f}{U}$.

Usage

```
reducedFrequency(wingSpan, frequency, speed)
```

Arguments

wingSpan Tip-to-tip distance of the fully spread wing (m)

frequency	Wingbeat frequency (1/s)
speed	Airspeed (m/s)

Details

This parameter is the ratio of the wingspan to the wavelength of the convected wake. For very high reduced frequencies, the wake of one wingbeat is relatively short compared to the wingspan, meaning that previous wingbeats have a large influence on the aerodynamics of the current wingbeat. When the reduced frequency is low, there is relatively little interaction between the wingbeats.

This wingspan based reduced frequency should not be confused with the chord based (or half chord) based reduced frequency. That definition serves a similar function, however, it relates to the effect of unsteadiness on the aerofoil (i.e. it is somewhat like the 2D equivalent).

Another related parameter of unsteadiness, often mentioned in relation to animal flight, is the Strouhal number, representing the ratio of the amplitude of the wingbeat to the wavelength of the wake. This term is historically related to vortex shedding.

Value

Numeric value

Author(s)

Marco Klein Heerenbrink

References

Klein Heerenbrink, M., Johansson, L. C. and Hedenström, A. 2015 Power of the wingbeat: modelling the effects of flapping wings in vertebrate flight. *Proc. R. Soc. A* **471**, 2177 doi:10.1098/rspa.2014.0952

See Also

[computeFlappingPower](#)

Examples

```
kf <- reducedFrequency(  
  wingSpan = 0.67,  
  frequency = 4,  
  speed = 9  
)  
kf  
# [1] 1.870993
```

Index

- * **datasets**
 - climbing_birds, [7](#)
- * **energy optimization**
 - findMinimumTimeSpeed, [26](#)
- * **migration**
 - findMinimumTimeSpeed, [26](#)

- air2ground, [2](#)
- altitude2density, [3](#)
- amplitude, [4](#), [15](#), [23](#)

- Bird, [5](#), [8–13](#), [15](#), [17](#), [21](#), [22](#), [24–26](#), [28](#)

- chem2mech, [12](#)
- chem2mech (PowerToFroMechChem), [28](#)
- climbing_birds, [7](#)
- computeAvailablePower, [7](#), [8](#)
- computeBodyFrontalArea, [7](#), [10](#)
- computeChemicalPower, [7](#), [11](#), [24](#), [27](#), [28](#)
- computeFlappingPower, [4](#), [6](#), [7](#), [12](#), [12](#), [17](#), [20–24](#), [26](#), [27](#), [30](#)
- computeFlightPerformance, [16](#)

- fD.ind, [15](#)
- fD.ind (fDfPfunctions), [19](#)
- fD.pro0, [15](#)
- fD.pro0 (fDfPfunctions), [19](#)
- fD.pro2, [15](#)
- fD.pro2 (fDfPfunctions), [19](#)
- fDfPfunctions, [19](#)
- findMaximumClimbRate, [21](#)
- findMaximumPowerSpeed, [22](#)
- findMaximumRangeSpeed, [24](#), [27](#)
- findMinimumPowerSpeed, [25](#)
- findMinimumTimeSpeed, [26](#)
- fP.ind, [15](#)
- fP.ind (fDfPfunctions), [19](#)
- fP.pro0, [15](#)
- fP.pro0 (fDfPfunctions), [19](#)
- fP.pro2, [15](#)
- fP.pro2 (fDfPfunctions), [19](#)

- mech2chem, [12](#)
- mech2chem (PowerToFroMechChem), [28](#)

- optimize, [26](#)

- PowerToFroMechChem, [28](#)

- reducedFrequency, [29](#)

- uniroot, [22](#), [23](#)