

Package ‘clime’

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Type Package

Title Constrained L1-Minimization for Inverse (Covariance) Matrix Estimation

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Depends lpSolve

Description A robust constrained L1 minimization method for estimating a large sparse inverse covariance matrix (aka precision matrix), and recovering its support for building graphical models. The computation uses linear programming. The method was published in TT Cai, W Liu, X Luo (2011) <[doi:10.1198/jasa.2011.tm10155](https://doi.org/10.1198/jasa.2011.tm10155)>.

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clime *solve for the inverse matrix*

Description

Solve for a series of the inverse covariance matrix estimates at a grid of values for the constraint lambda.

Usage

```
clime(x, lambda=NULL, nlambda=ifelse(is.null(lambda),100,length(lambda)),
      lambda.max=0.8, lambda.min=ifelse(nrow(x)>ncol(x), 1e-4, 1e-2),
      sigma=FALSE, perturb=TRUE, standardize=TRUE, logspaced=TRUE,
      linsolver=c("primaldual", "simplex"), pdtol=1e-3, pdmaxiter=50)
```

Arguments

x	Input matrix of size n (observations) times p (variables). Each column is a variable of length n. Alternatively, the sample covariance matrix may be set here with the next option sigma set to be TRUE. When the input is the sample covariance matrix, cv.clime can not be used for this object.
lambda	Grid of non-negative values for the constraint parameter lambda. If missing, nlambda values from lambda.min to lambda.max will be generated.
standardize	Whether the variables will be standardized to have mean zero and unit standard deviation. Default TRUE.
nlambda	Number of values for program generated lambda. Default 100.
lambda.max	Maximum value of program generated lambda. Default 0.8.
lambda.min	Minimum value of program generated lambda. Default $1e-4(n > p)$ or $1e-2(n < p)$.
sigma	Whether x is the sample covariance matrix. Default FALSE.
perturb	Whether a perturbed Sigma should be used or the positive perturbation added if it is numerical. Default TRUE.
logspaced	Whether program generated lambda should be log-spaced or linear spaced. Default TRUE.
linsolver	Whether primaldual (default) or simplex method should be employed. Rule of thumb: primaldual for large p, simplex for small p.
pdtol	Tolerance for the duality gap, ignored if simplex is employed.
pdmaxiter	Maximum number of iterations for primaldual, ignored if simplex is employed.

Details

A constrained ℓ_1 minimization approach for sparse precision matrix estimation (details in references) is implemented here using linear programming (revised simplex or primal-dual interior point method). It solves a sequence of lambda values on the following objective function

$$\min |\Omega|_1 \quad \text{subject to: } \|\Sigma_n \Omega - I\|_\infty \leq \lambda$$

where Σ_n is the sample covariance matrix and Ω is the inverse we want to estimate.

Value

An object with S3 class "clime". You can also use it as a regular R list with the following fields:

Omega	List of estimated inverse covariance matrix for a grid of values for lambda.
lambda	Actual sequence of lambda used in the program
perturb	Actual perturbation used in the program.
standardize	Whether standardization is applied to the columns of x.
x	Actual x used in the program.
lpfun	Linear programming solver used.

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References

Cai, T.T., Liu, W., and Luo, X. (2011). *A constrained ℓ_1 minimization approach for sparse precision matrix estimation*. Journal of the American Statistical Association 106(494): 594-607.

Examples

```
## trivial example
n <- 50
p <- 5
X <- matrix(rnorm(n*p), nrow=n)
re.clime <- clime(X)

## tridiagonal matrix example
bandMat <- function(p, k) {
  cM <- matrix(rep(1:p, each=p), nrow=p, ncol=p)
  return((abs(t(cM)-cM)<=k)*1)
}
## tridiagonal Omega with diagonal 1 and off-diagonal 0.5
Omega <- bandMat(p, 1)*0.5
diag(Omega) <- 1
Sigma <- solve(Omega)
X <- matrix(rnorm(n*p), nrow=n)%*%chol(Sigma)
re.clime <- clime(X, standardize=FALSE, linsolver="simplex")
```

```
re.cv <- cv.clime(re.clime)
re.clime.opt <- clime(X, standardize=FALSE, re.cv$lambdaopt)

## Compare Frobenius norm loss
## clime estimator
sqrt( sum( (Omega-re.clime.opt$Omegalist[[1]])^2 ) )
## Not run: 0.3438533
## Sample covariance matrix inversed
sqrt( sum( ( Omega-solve(cov(X)*(1-1/n)) )^2 ) )
## Not run: 0.874041
sqrt( sum( ( Omega-solve(cov(X)) )^2 ) )
## Not run: 0.8224296
```

clime-internal *internal clime functions*

Description

Internal clime functions

Usage

```
likelihood(Sigma, Omega)
tracel2(Sigma, Omega)
```

Arguments

Sigma	Covariance matrix.
Omega	Inverse covariance matrix.

Details

There are not intended for use by users.

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References

Cai, T.T., Liu, W., and Luo, X. (2011). *A constrained ℓ_1 minimization approach for sparse precision matrix estimation*. Journal of the American Statistical Association 106(494): 594-607.

cv.clime	<i>k-fold cross validation for clime object</i>
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Description

Perform a k-fold cross validation for selecting lambda

Usage

```
cv.clime(clime.obj, loss=c("likelihood", "tracel2"), fold=5)
```

Arguments

clime.obj	clime object output from clime. Note that this requires that the input to clime is x instead of the sample covariance matrix.
loss	loss to be used in cross validation. Currently, two losses are available: "likelihood" and "tracel2". Default "likelihood".
fold	number of folds used in cross validation. Default 5.

Details

Perform a k-fold cross validation for selecting the tuning parameter lambda in clime. Two losses are implemented currently:

$$\text{likelihood: } Tr[\Sigma\Omega] - \log |\Omega| - p$$

$$\text{tracel2: } Tr[\text{diag}(\Sigma\Omega - I)^2].$$

Value

An object with S3 class "cv.clime". You can use it as a regular R list with the following fields:

lambdaopt	the lambda selected by cross validation to minimize the loss over the grid values of lambda.
loss	the name of loss used in cross validation.
lambda	sequence of lambda used in the program.
loss.mean	average k-fold loss values for each grid value lambda.
loss.mean	standard deviation of k-fold loss values for each grid value lambda.
lpfun	Linear programming solver used.

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References

Cai, T.T., Liu, W., and Luo, X. (2011). *A constrained ℓ_1 minimization approach for sparse precision matrix estimation*. Journal of the American Statistical Association 106(494): 594-607.

Examples

```
## trivial example
n <- 50
p <- 5
X <- matrix(rnorm(n*p), nrow=n)
re.clime <- clime(X)
re.cv <- cv.clime(re.clime)
re.clime.opt <- clime(X, re.cv$lambdaopt)
```

```
print.clime          print a clime object
```

Description

Print a summary of the clime object.

Usage

```
## S3 method for class 'clime'
print(x,digits = max(3, getOption("digits") - 3), ... )
```

Arguments

x	clime object.
digits	significant digits in printout.
...	additional print options.

Details

This call simply outlines the options used for computing a clime object.

Value

The output above is invisibly returned.

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References

Cai, T.T., Liu, W., and Luo, X. (2011). *A constrained ℓ_1 minimization approach for sparse precision matrix estimation*. Journal of the American Statistical Association 106(494): 594-607.

Examples

```
## trivial example
n <- 50
p <- 5
X <- matrix(rnorm(n*p), nrow=n)
re.clime <- clime(X)
print(re.clime)
```

print.cv.clime *print a cross validated clime object*

Description

Print a summary of the cv.clime object.

Usage

```
## S3 method for class 'cv.clime'
print(x,digits = max(3, getOption("digits") - 3), ... )
```

Arguments

x	cv.clime object.
digits	significant digits in printout.
...	additional print options.

Details

This call outputs first a three column matrix with lambda, mean and sd for the cross validation loss values. The actual loss used and the optimal lambda value picked by cv are printed.

Value

The output above is invisibly returned.

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References

Cai, T.T., Liu, W., and Luo, X. (2011). *A constrained ℓ_1 minimization approach for sparse precision matrix estimation*. Journal of the American Statistical Association 106(494): 594-607.

Examples

```
## trivial example
n <- 50
p <- 5
X <- matrix(rnorm(n*p), nrow=n)
re.clime <- clime(X)
re.cv <- cv.clime(re.clime)
print(re.cv)
```

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