

# Package ‘MBBEFDLite’

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**Type** Package

**Title** Statistical Functions for the  
Maxwell-Boltzmann-Bose-Einstein-Fermi-Dirac (MBBEFD) Family of  
Distributions

**Version** 0.0.5

**Description** Provides probability mass, distribution, quantile, random variate  
generation, and method-of-moments parameter fitting for the MBBEFD family of  
distributions used in insurance modeling as described in Bernegger (1997)  
<[doi:10.2143/AST.27.1.563208](https://doi.org/10.2143/AST.27.1.563208)> without any external dependencies.

**License** MPL-2.0

**Encoding** UTF-8

**Imports** stats

**Suggests** tinytest, covr

**URL** <https://github.com/aadler/MBBEFDLite>

**BugReports** <https://github.com/aadler/MBBEFDLite/issues>

**ByteCompile** yes

**NeedsCompilation** yes

**UseLTO** yes

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## Contents

MBBEFDLite-package . . . . .	2
dmb . . . . .	3
ecmb . . . . .	4
mommb . . . . .	6
<b>Index</b>	<b>8</b>

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MBBEFDLite-package	<i>Statistical Functions for the Maxwell-Boltzmann-Bose-Einstein-Fermi-Dirac (MBBEFD) Family of Distributions</i>
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## Description

Provides probability mass, distribution, quantile, random variate generation, and method-of-moments parameter fitting for the MBBEFD family of distributions used in insurance modeling as described in Bernegger (1997) <doi:10.2143/AST.27.1.563208> without any external dependencies.

## Details

The DESCRIPTION file:

```
Package:      MBBEFDLite
Type:         Package
Title:        Statistical Functions for the Maxwell-Boltzmann-Bose-Einstein-Fermi-Dirac (MBBEFD) Family of Dis
Version:      0.0.5
Authors@R:    person(given="Avraham", family="Adler",role=c("aut", "cre", "cph"), email="Avraham.Adler@gmail.c
Description:   Provides probability mass, distribution, quantile, random variate generation, and method-of-moments pa
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Maintainer:   Avraham Adler <Avraham.Adler@gmail.com>
Archs:        x64
```

Index of help topics:

MBBEFDLite-package	Statistical Functions for the Maxwell-Boltzmann-Bose-Einstein-Fermi-Dirac (MBBEFD) Family of Distributions
dmb	The MBBEFD Distribution
ecmb	Exposure Curve for the MBBEFD Distribution
mommb	Method of Moments Parameter Estimation for the MBBEFD distribution

## Author(s)

Avraham Adler [aut, cre, cph] (ORCID: <https://orcid.org/0000-0002-3039-0703>)  
 Maintainer: Avraham Adler <Avraham.Adler@gmail.com>

## Description

Density, distribution function, quantile function and random generation for the MBBEFD distribution with parameters  $g$  and  $b$ .

## Usage

```
dmb(x, g, b, c = NULL, log = FALSE)
pmb(q, g, b, c = NULL, lower.tail = TRUE, log.p = FALSE)
qmb(p, g, b, c = NULL, lower.tail = TRUE, log.p = FALSE)
rmb(n, g, b, c = NULL)
```

## Arguments

$x, q$	<b>numeric</b> ; vector of quantiles.
$p$	<b>numeric</b> ; vector of probabilities.
$n$	<b>numeric</b> ; number of observations. If $\text{length}(n) > 1$ , the length is taken to be the number required.
$g$	<b>numeric</b> ; (vector of) the $g$ parameter, which is also the reciprocal of the probability of a maximum loss.
$b$	<b>numeric</b> ; (vector of) the $b$ parameter.
$c$	<b>numeric</b> ; (vector of) the optional $c$ parameter. Should be <code>NULL</code> if $g$ and $b$ are passed. Otherwise, $g = e^{(0.78+0.12c)c}$ and $b = e^{3.1-0.15(1+c)c}$ .
$\log, \log.p$	<b>logical</b> ; if <code>TRUE</code> , probabilities $p$ are given as $\log(p)$ .
<code>lower.tail</code>	<b>logical</b> ; if <code>TRUE</code> (default), probabilities are $P[X \leq x]$ otherwise $P[X > x]$ .

## Details

The MBBEFD class of curves are defined in Bernegger (1997) and are often used to model insurance risk. The density is defined on the semi-open interval  $[0, 1)$  and the distribution and quantile functions are defined on the closed interval  $[0, 1]$ .

## Value

`dnorm` gives the density, `pnorm` gives the distribution function, `qnorm` gives the quantile function, and `rnorm` generates random deviates.

The length of the result is determined by  $n$  for `rnorm`, and is the length of  $x$ ,  $p$ , or  $q$  as appropriate for the other functions.

Numerical arguments other than  $n$  are recycled to the length of the result. Logical arguments should be of length 1.

**Note**

This package follows Bernegger's convention that the density function does not exist at 1. This differs from the **mbbefd** package.

**Author(s)**

Avraham Adler <Avraham.Adler@gmail.com>

**References**

Bernegger, S. (1997) The Swiss Re Exposure Curves and the MBBEFD Distribution Class. *ASTIN Bulletin* **27**(1), 99–111. doi:[10.2143/AST.27.1.563208](https://doi.org/10.2143/AST.27.1.563208)

**See Also**

[mommb](#) for parameter estimation.

**Examples**

```
all.equal(dmb(0.5, 1, 0), 0)
dmb(0.2, 20, 5)
pmb(0.98, 25, 4)
qmb(0.98, 25, 4) == 1
all.equal(qmb(pmb(0.98, 25, 4), 25, 4), 0.98)
set.seed(45)
rmb(3, 4, 12)
set.seed(45)
rmb(99:101, 4, 12) # Should equal previous call
```

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ecmb

*Exposure Curve for the MBBEFD Distribution*

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**Description**

Returns the limited average severity at  $x$  of a random variable with an MBBEFD distribution with parameters  $g$  and  $b$ .

**Usage**

```
ecmb(x, g, b, c = NULL, lower.tail = TRUE)
```

**Arguments**

$x$	<b>numeric</b> ; vector of quantiles.
$g$	<b>numeric</b> ; (vector of) the $g$ parameter, which is also the reciprocal of the probability of a maximum loss.
$b$	<b>numeric</b> ; (vector of) the $b$ parameter.

- c** **numeric**; (vector of) the optional **c** parameter. Should be NULL if **g** and **b** are passed. Otherwise,  $g = e^{(0.78+0.12c)c}$  and  $b = e^{3.1-0.15(1+c)c}$ .
- lower.tail** **logical**; if TRUE (default), percentages are of the total loss being less than or equal to **x**. Otherwise they are the percentage of total loss greater than **x**.

### Details

Given random variable  $X$  with an MBBEFD distribution with parameters  $g$  and  $b$ , the **exposure curve** (EC) is defined as the ratio of the limited average severity (LAS) of the variable at  $x$  divided by the unlimited expected severity of the distribution:

$$EC(x) = \frac{LAS(x)}{E(X)} = \frac{E(X \wedge x)}{E(X)} = \frac{\int_0^x tf(t)dt + x \int_x^\infty f(t)dt}{\int_0^\infty tf(t)dt}$$

If one considers  $x$  as a policy limit, then the value of `ecmb(x, g, b)` is the percentage of the total expected loss which will be contained within the (reinsurance) policy limit. If `lower.tail` is FALSE, the return value is the percentage of total loss which will exceed the limit.

### Value

A numeric vector containing the values of the exposure curve for the passed **x**, **b**, and **g** vectors. If `lower.tail` is FALSE, the return value is the complement of the exposure curve.

### Author(s)

Avraham Adler <Avraham.Adler@gmail.com>

### References

Bernegger, S. (1997) The Swiss Re Exposure Curves and the MBBEFD Distribution Class. *ASTIN Bulletin* **27**(1), 99–111. doi:[10.2143/AST.27.1.563208](https://doi.org/10.2143/AST.27.1.563208)

### See Also

[dmb](#) and [pmb](#) for the density and distribution.

### Examples

```
all.equal(ecmb(c(0, 1), 20, 5), c(0, 1))
ecmb(0.25, 100, 34)
```

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mommb	<i>Method of Moments Parameter Estimation for the MBBEFD distribution</i>
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## Description

Attempts to find the best  $g$  and  $b$  parameters which are consistent with the first and second moments of the supplied data.

## Usage

```
mommb(x, maxit = 100L, tol = .Machine$double.eps ^ 0.5, na.rm = TRUE)
```

## Arguments

<code>x</code>	<b>numeric</b> ; vector of observations between 0 and 1.
<code>maxit</code>	<b>integer</b> ; maximum number of iterations.
<code>tol</code>	<b>numeric</b> ; tolerance. If too tight, algorithm may fail. Defaults to the square root of <code>.Machine\$double.eps</code> or roughly $1.49 \times 10^{-8}$ .
<code>na.rm</code>	<b>logical</b> ; if TRUE (default) NAs are removed. If FALSE, and there are NAs, the algorithm will stop with an error.

## Details

The algorithm is based on sections 4.1 and 4.2 of Bernegger (1997). With rare exceptions, the fitted  $g$  and  $b$  parameters must conform to:

$$\mu = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$$

subject to:

$$\mu^2 \leq E[x^2] \leq \mu p \leq E[x^2]$$

where  $\mu$  and  $\mu^2$  are the “true” first and second moments and  $E[x^2]$  is the empirical second moment.

The algorithm starts with the estimate  $p = E[x^2]$  as an upper bound. However, in step 2 of section 4.2, the  $p$  component is estimated as the difference between the numerical integration of  $x^2 f(x)$  and the empirical second moment— $p = E[x^2] - \int x^2 f(x) dx$ —as seen in equation (4.3). This is converted to  $g$  by reciprocation and convergence is tested by the difference between this new  $g$  and its prior value. If the new  $p \leq 0$ , the algorithm attempts to restart with a larger  $g$ —a smaller  $p$ . In this case, the algorithm tends to fail to converge.

**Value**

Returns a [list](#) containing:

<code>g</code>	The fitted <code>g</code> parameter.
<code>b</code>	The fitted <code>b</code> parameter.
<code>iter</code>	The number of iterations used.
<code>sqerr</code>	The squared error between the empirical mean and the theoretical mean given the fitted <code>g</code> and <code>b</code> .

**Note**

Anecdotal evidence indicates that the results of this fitting algorithm can be volatile, especially with fewer than a few hundred observations.

**Author(s)**

Avraham Adler <Avraham.Adler@gmail.com>

**References**

Bernegger, S. (1997) The Swiss Re Exposure Curves and the MBBEFD Distribution Class. *ASTIN Bulletin* **27**(1), 99–111. doi:[10.2143/AST.27.1.563208](https://doi.org/10.2143/AST.27.1.563208)

**See Also**

[rmb](#) for random variate generation.

**Examples**

```
set.seed(85L)
x <- rmb(1000, 25, 4)
mommb(x)
```

# Index

## \* **distribution**

dmb, [3](#)

ecmb, [4](#)

MBBEFDLite-package, [2](#)

mommb, [6](#)

## \* **package**

MBBEFDLite-package, [2](#)

dmb, [3](#), [5](#)

ecmb, [4](#)

list, [7](#)

MBBEFDLite-package, [2](#)

mommb, [4](#), [6](#)

pmb, [5](#)

pmb (dmb), [3](#)

qmb (dmb), [3](#)

rmb, [7](#)

rmb (dmb), [3](#)